A Legendre Polynomial Solver for the Langevin Boltzmann Equation

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Due to the proliferation of wireless and other RF applications, noise simulation has become an important topic of TCAD. Although the so-called physical Monte Carlo (MC) method inherently contains electronic noise, this time-domain based method is far too slow for most noise calculations, which are performed in the GHz range or below, because the CPU time is at least inversely proportional to the minimum frequency investigated. On the other hand, the Langevin Boltzmann equation (LBE), which is the basis of the MC method, can be also solved directly in the frequency domain by other numerical methods, thus avoiding the CPU time increase at low frequencies. This is demonstrated for the first time in this paper.

The steady-state distribution function and the small-signal Green's functions are expanded into Legendre Polynomials up to the nth order together with the LBE and the spectral intensity of the Langevin force, where the energy domain is discretized with finite differences. Electron transport in Si is modeled with Jacoboni's anisotropic and nonparabolic six valley model, which can be easily extended to the case of strained Si and SiGe.

In Fig. 1 the longitudinal and transverse components of the diffusion tensor ($\tau v v^T$) are shown for electrons in undoped Si at room temperature as a function of an electric field in the <100> direction. This transport parameter, which is related to the noise source of the velocity fluctuations, contains the dyadic product of the velocity and therewith the Legendre polynomial of the second order. Thus, an expansion up to only the first order, as is often done, is not sufficient, and large errors are encountered when only the first order is considered (Fig. 1). On the other hand, expansions higher then the third order yield only negligible improvements (not shown) and good agreement with MC results is obtained already with the third order expansion. In Fig. 2 the spectral intensity of the velocity fluctuations is shown for 0Hz, and the differences between first and third order expansions are astonishingly small. Larger differences are found for higher frequencies (Fig. 3). Again, excellent agreement with MC results is obtained. In Fig. 4 the spectral intensity of the cross correlation of energy and velocity fluctuations is shown, where the imaginary part vanishes for small frequencies. Such quantities are notoriously difficult to evaluate by MC simulation. The MC CPU time increases roughly by a factor of 1000 for a decrease in frequency by 10. Therefore, at frequencies below about 100GHz the presented numerical approach is many orders of magnitude faster than MC. But even for the data shown in Fig. 3 the CPU time of the numerical solver is about 100 times lower than MC. In devices, where slow processes like generation/recombination are important, the new numerical approach is the only feasible way to solve the LBE in the RF range or below.

Conclusions: We have presented the first numerical solver for the LBE in the frequency domain, which was successfully verified by comparison with MC results. It was found that noise calculation requires a Legendre Polynomial expansion up to the third order. Nevertheless, the new method is orders of magnitude faster than corresponding MC simulations.

A full journal publication of this work will be published in the Journal of Computational Electronics.

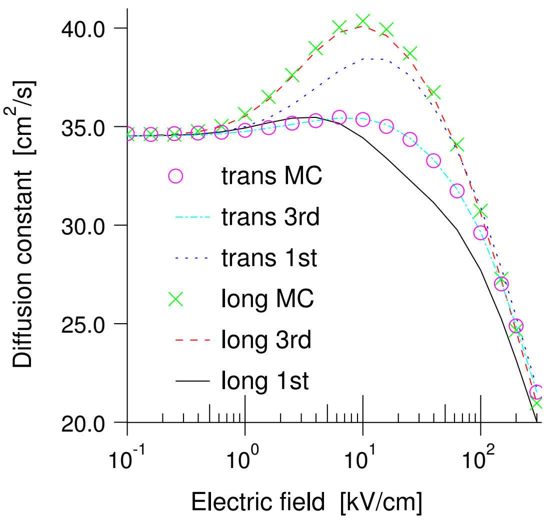

Fig. 1 Longitudinal and transverse diffusion constant obtained by first and third order Legendre Polynomial expansions.

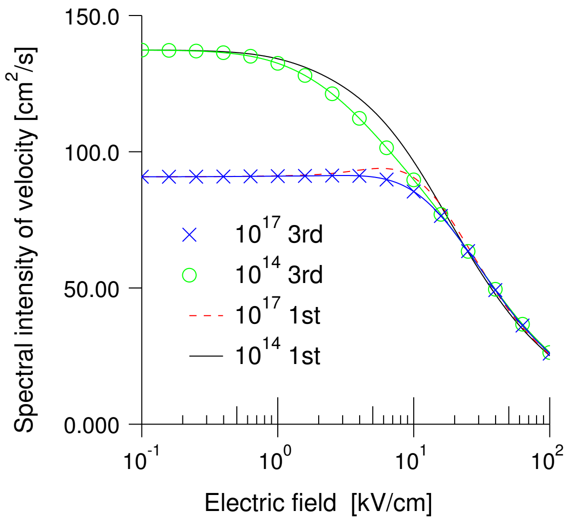

Fig. 2 Spectral intensity of the longitudinal velocity fluctuations for an n-type doping of $10^{14}/cm^3$ and $10^{17}/cm^3$ at 0Hz.

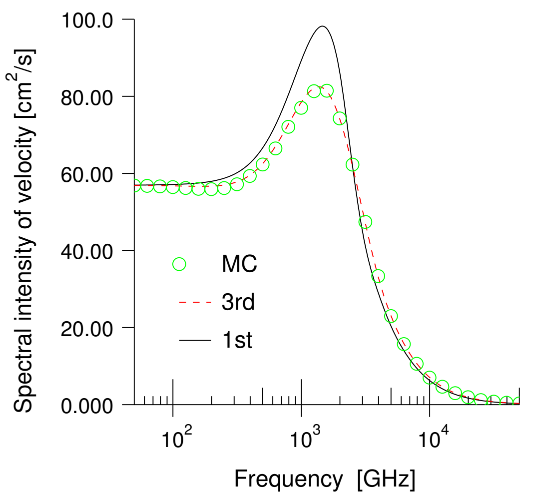

Fig. 3 Spectral intensity of the longitudinal velocity fluctuations for an electric field of 30kV/cm and undoped Si.

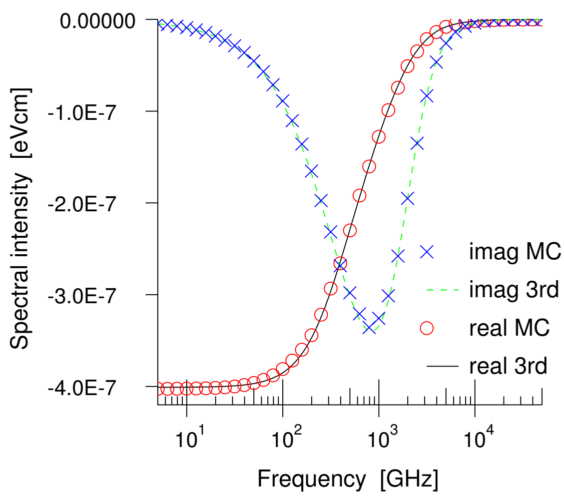

Fig. 4 Real and imaginary part of the spectral cross power of the longitudinal velocity and energy fluctuations for an electric field of 30kV/cm.

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